Homework 2 on 22 — Solutions — Meyer Alvo

(1) Ch 2 p 48 #1 a) $S = \{(r,r), (r,g), (r,b) (g,r) (g\ g) (g,b)$
$(b,r), (b,g), (b,g)\}$

(1) b) $S = \{(r,g) (r\ b), (g,r) (g,b) (b\ r) (b\ g)\}$

(1) p48 #5 a) 2 choices each time; hence $2^5 = 32$

(1) b). Set 1 if components are working and 0 otherwise. Consider a 5-dimensional vector $(x_1, \ldots, x_5)$ where $x_i = \begin{cases} 1 & \text{if component } i \text{ works} \\ 0 & \text{— } i \text{ fails} \end{cases}$

$$W = \{(1,1,1,1,1) (1,1,1,1,0) (1,1,1,0,1) (1,1,0,1,1)$$
$$(1,1,1,0,0) (1,1,0,1,0) (1,1,0,0,1) (1,1,0,0,0)$$
$$(1,0,1,1,1) (0,1,1,1,1) (1,0,1,1,0) (0,1,1,1,0)$$
$$(0,0,1,1,1) (0,0,1,1,0) (1,0,1,0,1)\}$$

(1) c) A contains 8 outcomes i.e. $x_4 = x_5 = 0$ but x_1, x_2, x_3 can each take 2 values 0 or 1. Hence $2^3 = 8$

(1) d) $A \cap W = \{(1,1,1,0,0) (1,1,0,0,0)\}$

(1) p48 #7 a) For each we have a choice of 2 (blue collar, white collar) and a choice of 3 (Rep., Demo., Indep.) Hence $(2)(3) = 6$ for each of 15 individuals.
$\Rightarrow 6^{15}$

(1) b) We consider the complementary event that none are blue collar workers $\Rightarrow 3^{15}$ and hence for at least one we have $6^{15} - 3^{15}$

(1) c) Clearly 4^{15}

(1) p48 #8 a) $P(A \cup B) = P(A) + P(B) = 0.3 + 0.5 = 0.8$

(1) b) $P(A \cap B^c) = P(A) = 0.3$

(1) c) $P(A \cap B) = 0$

p49 #15 We use the display

Suits

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & & & & \\ 2 & & & & \\ \cdot & & & & \\ \cdot & & & & \\ 10 & & & & \\ J & & & & \\ Q & & & & \\ K & & & & \end{array}$$

Face Value

(2) a) For a flush, we choose the suit in $\binom{4}{1}$ ways and 5 cards in $\binom{13}{5}$

$\therefore$ Prob(flush) $= \binom{4}{1}\binom{13}{5} / \binom{52}{5}$

(2) b) For one pair, we choose a row in $\binom{13}{1}$ ways and two cards from that row in $\binom{4}{2}$ ways.
Next we choose 3 rows in $\binom{12}{3}$ ways and one card from each row in 4^3

$\therefore$ Prob.(pair) $= 13 \binom{4}{2}\binom{12}{3} 4^3 / \binom{52}{5}$

(2) c) For two pairs we choose 2 rows in $\binom{13}{2}$ ways and one card from each in $\binom{4}{2}^2$ ways. We choose the last card in $\binom{44}{1}$ ways

$\therefore$ Prob(two pairs $= \binom{13}{2}\binom{4}{2}^2\binom{44}{1} / \binom{52}{5}$

(2) d) For three of a kind, we choose a row in $\binom{13}{1}$ ways and then 3 cards from that row in $\binom{4}{3}$ ways. Next we choose two other rows in $\binom{12}{2}$ ways and one card from each in $\binom{4}{1}^2$

$\therefore$ Prob(three of a kind) $= 13\binom{4}{3}\binom{12}{2} 4^2 / \binom{52}{5}$

(2) e) This is $\binom{13}{1}\binom{4}{4}\binom{48}{1} / \binom{52}{5}$